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U.C.C. CONFERENCE 1989

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JOEL ANDERSON: Approximation by extreme points.

__Notation__: If $p \in M_n(\mathbb{C})$ is a projection, $p = p^* = p^2$ with rank $(p) = r$,

$2r < n$ and $p = [\pi_{ij}]$, $\delta = \text{Max } \{\pi_{ii} : 1 \leq i \leq n\}$.

__Conjecture 1__: There exists $\epsilon > 0$ depending only on δ such that given p as

above, there exists a diagonal projection $g = g_\sigma = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & 1 \\ 0 & & -1 \end{bmatrix}$ such that

$$\|gpg + (1-g)p(1-g)\| \leq 1-\epsilon.$$

__Prop.__: Conjecture 1 is equivalent to the following.

If $a_1, \ldots, a_n \in M_r(\mathbb{C})$, $0 \leq a_i \leq \delta$, $a_1 + \ldots + a_n = 1_r$

then there is $\sigma \subseteq \{1, 2, \ldots, n\}$ such that

$$\left\| \sum_{i \in \sigma} a_i - \frac{1}{2} 1_r \right\| \leq \frac{1}{2} - \epsilon.$$

__Theorem__: If $\psi : M \to \ell^\infty(\{1, \ldots, r\})$

is a linear map of the von Neumann algebra M, $K = \{x \in M : 0 \leq x \leq 1\}$
and $S = \sup\{\|\psi(g)\|_1 : g$ is a minimal projection in $M\}$ then given $c \in K$ there

is an extreme point $e \in K$ such that
$$\|\psi(e) - \psi(c)\|_\infty \leq 2S.$$

If ψ is self-adjoint, we have
$$\|\psi(e) - \psi(c)\|_\infty \leq S.$$

R.J. ARCHBOLD: More crossed product structure in the Cuntz algebra

The Cuntz algebra O_2 is the simple, nuclear C*-algebra generated by two isometries S_1 and S_2 such that $S_1 S_1^* + S_2 S_2^* = 1$. This algebra contains a canonical copy of the Fermion algebra F_2 which, in turn, contains a canonical diagonal maximal abelian subalgebra D. Cuntz showed that O_2 may be regarded as a crossed product of F_2 by a single endomorphism. Since F_2 is a crossed product of D by a group action, Evans was able to show that $O_2 \otimes K$ (K the compact operators) is a crossed product of an abelian algebra closely related to D, the action being given by a semi-direct product group.

We make the observation that O_2 may also be regarded as the crossed product of D by the group G generated by the Choi unitaries $U = S_1 S_2^* + S_2 S_1^*$ and $V = S_1 S_2^* S_2^* + S_2 S_2 S_1^* S_2^* + S_2 S_1 S_1^*$ (which satisfy $U^2 = V^3 = 1$). Elements of G act on D by conjugation and the action is faithful and ergodic. However, D has no G-invariant state.

Since D is abelian, G also acts as a group of homeomorphisms $\{\phi_w : w \in G\}$ on the maximal ideal space $X = \prod_1^\infty \mathbb{Z}_2$ of D. One may check directly that this action is minimal and that if $w \neq 1$ then the interior of the fixed point set X^{ϕ_w} is empty (these facts also follow from the simplicity of O_2 by a general result of Kawamura).

LAURA BURLANDO: Continuity points of the spectrum and spectral radius
functions.

In a paper to appear we give the following sufficient condition for
continuity of the spectral radius function r at a point a of a complex Banach
algebra L with identity e. If $\sigma(a)$ denotes the spectrum of a, let S(a)
denote the set of all points $\lambda \in \sigma(a)$ such that $\lambda e-a$ belongs to the interior
of non-invertible elements of L. We also set $\gamma(a) = \sup \{|\lambda|:\lambda\in S(a)\}$ and,
for any $J \in J_L$ (where J_L denotes the set of all proper closed two-sided
ideals of L), $\sigma_J(a) = \sigma(a+J)$ and $r_J(a) = \sup \{\inf\{|\lambda|:\lambda\in\omega\}:\omega$ is a component
of $\sigma_J(a)\}$. Then

Theorem 1

If $a \in L$ is such that $r(a) = \gamma(a)\vee(\sup\{r_J(a): J \in J_L\})$, r is continuous
at a.

In another paper to appear we give the following sufficient condition
for continuity of the spectrum function σ at $a \in L$. Let $\varsigma(a)$ denote the set
of all points $\lambda \in \sigma(a)$ such that any neighbourhood of λ contains a component
of $\sigma_J(a)$ for some $J \in J_L$. Then

Theorem 2

If $a \in L$ is such that $\sigma(a) = cl(S(a))\cup\varsigma(a)$, σ is continuous at a.

MAN-DUEN CHOI: The Cantor spectrum of some almost periodic Schrödinger
Operators.

The study of almost periodic Schrödinger operators has aroused great
interest in recent research. There arise many intriguing results connecting
operator theory, harmonic analysis, and difference equations. We will
exhibit two hermitian operators A and B such that the spectrum of each
operator is an interval, but the spectrum of A + rB is a Cantor set for each
non-zero real r.

MAN-DUEN CHOI: Representations of Projections.

How to locate the generic positions for two, three, or more linear
subspaces of a Hilbert space? We propose an intensive study on the
representation theory of finitely many projections. This leads to much
information about the affine structure of the family of contractive positive
operators. In particular, we show that each nxn contractive
positive-semi-definite matrix can be written as a convex combination of
$[\log_2 n] + 2$ projections (the number $[\log_2 n] + 2$ is sharp for each n).

E. EFFROS: A new approach to operator spaces.
(Joint work with Z.J. Ruan)

An <u>operator space</u> is any linear space of bounded operators on a Hilbert space H.   These were abstractly characterized by Ruan via their matricial norms.   We show that when provided with the appropriate matricial norms, the dual of an operator space is again an operator space.   We use the resulting modified theory to prove an operator analogue of Grothendieck's tensor characterization of the approximation property for normed spaces (i.e., function spaces).

E. EFFROS: Multivariable non-commutative Fourier analysis.
(Joint work with Z.J. Ruan)

We show how the Wittstock-Paulsen theory may be used to prove Haagerup's characterization of the completely bounded norm of multipliers, and how the Christensen-Sinclair theory may give multivariable analogues of this theory. We then indicate how the Haagerup tensor product may be used to concretely realize the operator dual of the n x n matrices.

SHAHN MAJID: Hopf-von Neumann Algebras Arising from Quantum Mechanics on
Homogeneous Spacetimes.

Two groups G_1, G_2 are said to be a *matched pair* if each acts on the space
of the other and these actions, (α, β) say, obey

$$\alpha_g(st) = \alpha_{\beta_t}(g)(s)\, \alpha_g(t), \quad \alpha_g(e) = e$$

$$\beta_s(gh) = \beta_{\alpha_h(s)}(g)\, \beta_s(h), \quad \beta_s(e) = e$$

$\forall\, g, h \in G_1$, $s, t, \in G_2$. Here e denotes the identity. To each locally
compact matched pair we associate a *bicrossproduct* Hopf-von Neumann algebra,
$M(G_1)^\beta\bowtie_\alpha L^\infty(G_2)$ as follows. As an algebra, it is the crossproduct von

Neumann algebra by the induced action of G_1 on $L^\infty(G_2)$. The coproduct is
defined by a dual cross coproduct construction obtained by twisting the
tensor product coproduct structure by β. The resulting $\overset{b_i}{\wedge}$crossproduct is both
non-commutative and non-cocommutative for non-trivial α, β.

If the modules for the actions α, β are also matched then these bicross-
products are Kac algebras with Haar weight given by the dual weight of the
Haar weight on $L^\infty(G_2)$. Then the dual Kac algebra is given by

$$(M(G_1)^\beta\bowtie_\alpha L^\infty(G_2))^\wedge = M(G_2)^\alpha\bowtie_\beta L^\infty(G_1).$$

The bicrossproducts arise as the algebra of observables of a quantum
system, that of a particle on a curved spacetime. The additional coproduct
structure encodes the dynamics of the particle and maintains symmetry between
observables and states. Examples exist with G_1 a simply-connected Lie group
and choices of G_2 determined by suitable solutions of the Classical
Yang-Baxter Equations on the complexification of the Lie algebra of G_1.

MARTIN MATHIEU: The C*-algebra of quotients

In this talk we discuss the three different possibilities of defining a
C*-algebra of quotients of a C*-algebra A with respect to a filter F of closed
essential ideals of A: the construction as a direct limit of multiplier
algebras, the approach via essentially defined double centralisers, and the
abstract characterisation. As a particular case we obtain a localised version
of the multiplier algebra. These notions may be motivated by the
well-established theory of (one-sided) rings of quotients for non-commutative
semi-prime rings and the more recent concept of a symmetric ring of quotients.
We give a characterisation of the primeness of a C*-algebra in terms of
non-commutativity of the local multiplier algebra as well as necessary
conditions for the completeness of the symmetric normed algebra of quotients
of a C*-algebra.

RICHARD MERCER: Periodic Unitaries in von Neumann Algebras.

Given a von Neumann algebra M and a Cartan subalgebra A, we consider
partial isometries in M which normalize A.   Such partial isometries are said
to act freely if they never act as the identity when restricted by a
projection in A, and they are said to be n-periodic if their domain and range
are equal and their nth power is the identity.

The following results are announced: For each n there exist
sufficiently many freely-acting n-periodic partial isometries to generate the
equivalence relation associated with the pair (M,A).   Each of these can be
extended to freely-acting n-periodic unitaries.   For both the partial
isometries and the unitaries all powers which are not multiples of n are
freely acting.   Finally, a sequence of periodic unitaries may be constructed
which converge to a unitary all of whose nonzero powers act freely.

These results are steps towards resolving the more difficult question
posed by Feldman and Moore of whether the equivalence relation associated
with the pair (M,A) is generated by a freely acting group.

RYSZARD NEST: Cyclic Cohomology of non-commutative tori.

We compute the cyclic cohomology of a non-commutative torus, i.e. a certain algebra Ω associated with an antisymmetric bicharacter of a finite rank free abelian group G.
The main result is

$$H^n(\Omega) = \text{Ker } S \dotplus \text{Im } S \dotplus V_n,$$

where

$$V_n \tilde{=} \Lambda^n(G \otimes \mathbb{C}).$$

The method of computation generalises the computation of the cyclic cohomology of the irrational rotation algebras given by Connes. Our method works equally well also in the rational case, which was dealt with by a different method by Connes.

JOHN LINDSAY ORR: Triangular Algebras and Ideals of Nest Algebras.

Triangular algebras are a class of non-self adjoint operator algebras which have motivated much study in that area. It is a long-standing question whether the compression of a maximal triangular algebra to the range of a projection in its core is always maximal triangular.   Providing a negative answer to this question motivates a study of ideals of nest algebras leading to the identification of Larson's ideal $\mathfrak{R}_N^\infty$ as the greatest diagonal-disjoint ideal of a nest algebra.   Techniques leading to this result were described.

GERT K. PEDERSEN: Real Rank of C*-algebras.

Based on ideas of M.A. Rieffel a notion of real rank was introduced, such that $RR (C(X)) = \dim (X)$ for every compact Hausdorff space. It was shown that $RR(A) \leq 2Bsr(A) - 1$ for any C*-algebra A, where Bsr means Bass stable rank (= topological stable rank), and that $RR(A) = 0$ for any von Neumann algebra A.

The case $RR(A) = 0$ was then discussed in some detail, and the condition was shown to be equivalent to previously defined conditions, known as (HP) and (FS). Finally a series of examples and open problems were mentioned, with lively participation from the audience.

R.J. PLYMEN: Arithmetic aspect of operator algebras.

Let C be a linear connected reductive Lie group, and let $C_r^*(G)$ denote its reduced C*-algebra. A. Wassermann has determined the structure of $C_r^*(G)$, up to stable isomorphism, as a direct sum of component C*-algebras: each component is the crossed product of an abelian C*-algebra by a finite group. Each finite group is the R-group of a discrete series representation of the ${}^0M$ subgroup of a Levi factor M.

Let now G be a p-adic Chevalley group. The basic example of a p-adic Chevalley group is the special linear group SL(n) over Q_p. We recall that a p-adic Chevalley group is semisimple. We have established that $C_r^*(G)$ is a direct sum of fixed-point algebras. Our main theorem is to give sufficient conditions under which the fixed-point algebras are stably isomorphic to the crossed product of an abelian unital C*-algebra by a finite group. Under these conditions, the conclusion is similar to the case of linear connected reductive Lie groups.

We consider the p-adic Chevalley group SL(ℓ) with ℓ prime and ℓ dividing p-1. In this case these are ℓ-1 component C*-algebras stably isomorphic to

$$C(T^\ell/T) \rtimes \mathbb{Z}/\ell$$

where $T = \left\{ z \in \mathbb{C} : |z| = 1 \right\}$, and $\mathbb{Z}/\ell$ acts by cyclic permutation. The ℓ fixed points have an arithmetic significance. Each fixed point σ is associated to a cyclic of order ℓ totally ramified extension field, and all such extension fields are accounted for in this way.

S.C. POWER: Subalgebras of AF C*-algebras.

A *canonical*, not necessarily self-adjoint, closed subalgebra A of an approximately finite C*-algebra B, is one for which there is a system B_n of finite-dimensional C*-algebras, increasing to B, such that for each n, $A_n = A \cap B_n$ contains a maximal abelian self-adjoint subalgebra of B_n. Equivalently A is the direct limit of finite-dimensional poset algebras with respect to embeddings $A_n \to A_{n+1}$ which have C*-extensions $C^*(A_n) \to C^*(A_{n+1})$. A cononical subalgebra is <u>triangular</u> if $A \cap A^*$ is a masa. We classify cononical triangular subalgebras in terms of the isomorphism class of the topological <u>fundamental relation</u> $(M(A \cap A^*), R(A))$, where M(.) indicates the Gelfand space of $A \cap A^*$ and where R(A) is the antisymmetric transitive reflexive relation on the Gelfand space induced by the partial isometries of A which normalise $A \cap A^*$. This leads to simple proofs of classicications of restricted classes considered by Baker (1987), Petors Porn and Wagner (1988), and Power (1988).

H. RADJAVI: On Reducibility of Semigroups of Compact Operators.

We consider conditions under which a (multiplicative) semigroup S of compact operators on a Hilbert space H is reducible, i.e., there is a closed subspace of H, other than {0} and the whole space, that is invariant under every member of S. Some, but by no means all, reducibility questions for semigroups can be answered by considering the corresponding questions for algebras. We discuss some of these problems and present some results.

A sample affirmative result: if the spectral radius r(.) is submultiplicative on S and if r(S) is positive and dominant in the spectrum of S for every S in S, then S is reducible. 'Dominant' means that r(S) is in the spectrum and every other point of the spectrum has absolute value less than r(S). Some corollaries are given; e.g., a semigroup of compact idempotents is triangularizable. A sample negative result: there is an irreducible semigroup every member of which is ±1 times an idempotent of rank one.

M. THELWALL: Bimodules in AF algebras.

Let B be an AF algebra.   Let C be a canonical masa in B, so that there
is a chain $B_1 \subseteq B_2 \subseteq \ldots$ say of finite-dimensional *-subalgebras of B with

$B_\infty = \bigcup_{n=1}^{\infty} B_n$, $C_n = C \cap B_n$, $C_\infty = \bigcup_{n=1}^{\infty} C_n$ and B_∞ is dense in B, C_n is a masa in B_n

and C_∞ is dense in C.

A C-bimodule is a closed subspace M of $\mathcal{B}$ such that $CMC \subseteq M$.   A result of
Power is that every C-bimodule M is the closed union of all the $M_n = M \cap B_n$, a
property we call inductivity.   The C-bimodules have much more structure than
just inductivity and together these properties allow much analysis from
finite dimensions to carry over.

We can always choose matrix units for each B_n so that C_n is spanned by
the matrix units in B_n which are projections.   The set of these matrix units
is called a system of matrix units for B.   The matrix units in a C-bimodule
define that bimodule and this information can be used to characterise a
bimodule M by its fundamental relation R(M).   R(M) is defined for all x and
y in the maximal ideal space of C by xR(M)y if there is a matrix unit v in M
such that $y(c) = x(vcv^*)$ for all c in C.

We illustrate this with two results.   If M is a unital algebra and the
underlying graph of R(M) is chordal then every contractive Hilbert space
representation of M $\underset{\wedge}{has}$ a Stinespring dilation.   This is a generalisation of
a similar result in finite dimensions.   Also, a characterisation of the
fundamental relations of maximal triangular algebras gives an example of such
an algebra A which does not contain any matrix units which sum to a fixed
matrix unit or its adjoint, so there is a large gap between A+A* and the AF
algebra A is maximal in.

RICHARD M. TIMONEY: Extremal mappings for the Schwarz lemma.
(Joint work with S. Dineen)

If $\mathfrak{D}_1$ and $\mathfrak{D}_2$ are balanced bounded pseudoconvex domains in Banach spaces X and Y, then we consider the largest $r \geq 0$ for which there is a holomorphic embedding $f:\mathfrak{D}_1 \to \mathfrak{D}_2$ such that $r\mathfrak{D}_2 \subset f(\mathfrak{D}_1)$. We show that in certain cases the embedding f must be linear, extending results of H. Alexander. If $\mathfrak{D}_1$ and $\mathfrak{D}_2$ are unit balls, $1/r$ is the Banach Mazur distance between X and Y.

For X and Y finite dimensional, we also consider the problem (which was posed originally by Carathéodory) of maximising $|\det f'(0)|$ over embeddings $f:\mathfrak{D}_1 \to \mathfrak{D}_2$. When $\mathfrak{D}_1$ is the unit ball of X and $\mathfrak{D}_2$ is the unit ball of a Hilbert space Y, we relate the problem to the minimal volume ellipsoid of F. John and give conditions which force f to be linear.

S. WASSERMANN: Nuclear embeddings and block diagonal operators.

A C*algebra is said to be <u>exact</u> if, for any short exact sequence
$0 \to I \to B \to C \to 0$ of C*algebras, the corresponding sequence of spatial tensor products

$$0 \to A \otimes I \to A \otimes B \to A \otimes C \to 0$$

is exact.

A is <u>nuclearly embeddable</u> if there is an isomorphic embedding $i : A \to B$ of A in a C*-algebra B and nets of contractive completely positive mappings $\{\sigma_\lambda\}$, $\{\tau_\lambda\}$, where $\sigma_\lambda : A \to M_{n_\lambda}(\mathbb{C})$, $\tau_\lambda : M_{n_\lambda}(\mathbb{C}) \to B$ such that $\tau_\lambda . \sigma_\lambda \to i$ in the point-norm topology.

If A is nuclearly embeddable, then it is exact.   If $A = C^*(G)$, the full C*-algebra of a discrete group G, then A is not exact when $G = F_n$, the free group on n generators, or when $G = SL_3(\mathbb{Z})$. It follows that, for these G, $C^*(G)$ is not nuclearly embeddable, and, more generally, that $C^*(G) \otimes M_n(\mathbb{C})$ is not for $n \geq 1$. $C^*(F_2) \otimes M_4(\mathbb{C})$ has a single generator T, and there is a faithful family of finite dimensional representations of this algebra.   It follows that T can be represented as a block-diagonal operator. on a Hilbert space which cannot be approximated in norm by operators which generate finite-dimensional C*-algebras.